

## TABLE OF CONTENTS

<b>ABOUT THE AUTHOR .....</b>	<b>10</b>
<b>LIST OF GRAPHICAL NOTATIONS AND SYMBOLS.....</b>	<b>12</b>
<b>PREFACE.....</b>	<b>13</b>
I.1. WHO WILL GAIN, AND WHAT, FROM THIS BOOK .....	13
I.4. ORGANIZATION OF THE BOOK .....	16
I.5. ACKNOWLEDGEMENTS.....	19
<b>CHAPTER 1.....</b>	<b>20</b>
<b>1. INTRODUCING INTEREST RATE.....</b>	<b>20</b>
<b>IRR EQUATION.....</b>	<b>20</b>
1.1. INTEREST RATE .....	21
1.1.1. Definition of interest rate .....	21
1.1.2. The meaning of negative power.....	27
1.1.3. Problems and Exercises: .....	28
1.2. INTRODUCING COMPOUNDING.....	30
1.2.1. Problems and Exercises: .....	33
1.3. DOMAINS OF INTEREST RATE AND PERIOD LENGTH .....	35
1.3.1. Specifics of negative interest rate .....	35
1.3.2. Can we use a real power in compounding formula?36	
1.3.3. The meaning of compounding when the period length is not an integer value.....	39
1.3.4. Problems and Exercises: .....	40
1.4. COMPUTING INTEREST RATE FOR DIFFERENT PERIOD LENGTHS.....	42
1.4.1. Relationship between the period length and interest42 rate .....	42
1.4.2. Computing interest rate for shorter or longer periods. Nominal and effective interest rates.....	44
1.4.3. Mathematical foundations of interest rate calculations .....	46
1.4.4. Computing interest rates. Numerical examples ....	49
1.4.5. Problems and Exercises: .....	52
1.5. CONTINUOUS COMPOUNDING.....	54
1.5.1. Continuous compounding. Definitions .....	55
1.5.2. Continuous compounding versus discrete compounding. Numerical example .....	59
1.5.3. Smooth exponential compounding function versus 62 the piecewise linear function .....	62
1.5.4. Problems and Exercises: .....	69
1.6. IRR EQUATION. AN INHERENT RELATIONSHIP OF COMPOUNDING OPERATION AND CASH FLOWS .....	72
1.6.1. How IRR equation accounts for cash transactions. 72 inside the period .....	72
1.6.2. Deriving simple form of IRR equation .....	74

1.6.3. Non-compounding scenario.....	78
1.6.4. Problems and Exercises: .....	79
1.7. DERIVING A GENERAL FORM OF IRR EQUATION .....	81
1.7.1. IRR Equation with discrete compounding.....	82
1.7.2. IRR equation with continuous compounding.....	84
1.7.3. Getting the “look and feel” of IRR equation.....	86
1.7.4. Problems and Exercises: .....	91
<b>CHAPTER 2.....</b>	<b>94</b>
<b>ANNUITIES.....</b>	<b>94</b>
2.1. DERIVING AN ORDINARY ANNUITY FROM THE IRR EQUATION.....	95
2.1.1. Annuity definition and specifics .....	95
2.1.3. Computing interest rate in case of annuities .....	100
2.1.5. Annuity’s present value .....	104
2.1.6. Problems and Exercises: .....	107
2.2. ANNUITIES AND WITHDRAWALS. PURCHASING AN ANNUITY.....	109
2.2.1. Accumulating required amount.....	111
2.2.2. Finding the number of annuity payments.....	113
2.2.3. Price of an annuity to receive regular payments .	115
2.2.4. Problems and Exercises: .....	116
2.3. DISCUSSION OF ANNUITY’S FEATURES .....	119
2.3.1. Annuity with a negative interest rate.....	119
Numerical example .....	119
2.4. ANNUITY DUE .....	125
2.4.1. Definition of annuity due .....	125
2.4.2. Deriving a general formula for an annuity due....	127
2.4.3. Finding regular payment .....	130
2.4.4. Finding initial payment .....	131
2.4.5. Determining the number of payments .....	132
2.4.6. Calculating the present value .....	133
2.4.7. Notes with regard to relationship between .....	135
annuities and IRR equation.....	135
2.4.8. Problems and Exercises: .....	136
2.5. ANNUITIES WITH CONTINUOUS COMPOUNDING.....	139
2.5.1. Ordinary annuity.....	139
2.5.2. Finding a regular payment.....	142
2.5.3. Calculating a lump sum payment .....	143
2.5.4. Finding the number of payments .....	145
2.5.5. Computing unknown interest rate .....	146
2.5.6. Finding the present value.....	147
2.1.7. Problems and Exercises: .....	148
2.6. RELATIONSHIP BETWEEN INTEREST RATES FOR PERIODS WITH DIFFERENT LENGTHS .....	151
2.6.1. Computing interest rate for an arbitrary period length .....	157
<b>CHAPTER 3.....</b>	<b>160</b>
<b>MORTGAGES. DERIVING MORTGAGE FORMULAS FROM IRR EQUATION.....</b>	<b>160</b>

3.1. DEFINING MORTGAGE IN MATHEMATICAL TERMS.....	162
3.1.1. Deriving a mortgage formula for remaining balance.....	162
3.1.2. Finding payment amount .....	165
3.1.3. Calculating number of periods .....	166
3.1.4. How much can we afford? .....	168
3.2. NUMERICAL EXAMPLES, GRAPHS AND DISCUSSION .....	170
3.2.1. Interest and principal amounts .....	170
3.2.2. Influence of payment frequency .....	173
3.2.3. Weekly and monthly payments versus annual payments.....	176
3.2.4. Problems and Exercises: .....	179
3.3. CHOOSING THE RIGHT METHOD TO CALCULATE AN INTEREST RATE FOR SHORTER OR LONGER PERIODS .....	182
3.3.1. Problems and Exercises: .....	185
3.4. GENERALIZATION OF MORTGAGE EQUATIONS.....	187
3.4.1. Proportionality between the period payment and principal amount .....	187
3.4.2. Relationship between the interest rate and period length .....	188
3.4.3. Problems and Exercises: .....	191
3.5. MORTGAGES WITH CONTINUOUS COMPOUNDING .....	193
3.5.1. Mortgage equations for continuous compounding.....	195
3.5.2. Finding the remaining balance .....	196
3.5.3. Calculating payment amount .....	197
3.5.4. Finding a number of payments .....	197
3.5.5. How much can we afford? .....	199
3.5.6. Problems and Exercises: .....	201
3.6. COMPARING MORTGAGES WITH DISCRETE AND CONTINUOUS COMPOUNDING.....	205
3.6.1. Problems and Exercises: .....	209
<b>CHAPTER 4.....</b>	<b>211</b>
<b>SOLVING NUMERICALLY MORTGAGE AND ANNUITIES EQUATIONS AS PARTICULAR CASES OF IRR EQUATION .....</b>	<b>211</b>
4.1. NEWTON-RAPHSON'S METHOD.....	213
4.1.1. Mathematical presentation.....	213
4.1.2. Problems and Exercises: .....	220
4.2. APPLICATION OF METHODS FOR SOLVING IRR EQUATION IN THE BUSINESS ENVIRONMENT .....	222
4.2.1. Choosing the first approximate value of interest rate for iterative procedures .....	223
4.2.2. Factors to be considered when adopting..... computational methods.....	224
4.2.3. Problems and Exercises: .....	225
<b>REFERENCES .....</b>	<b>227</b>
<b>INDEX.....</b>	<b>230</b>

