The role of conceptual context in the problem of finding rate of return

This article uncovers and explores fairly subtle issue at a first glance, namely compounding and non-compounding contexts in the problem of finding rate of return for investment portfolios. In reality this is a very important factor that should be considered as an essential characteristic of any method for calculating rate of return. IRR equation represents purely compounding context of application. The appropriate equation for non-compounding context has been derived. It turned out to be a flavor of Modified Dietz equation. Despite the mathematical similarity this equation was derived independently based only on assumption of non-compounding context of the problem finding rate of return. It also has some additional features so that we decided to name it as a *generalized Modified Dietz equation* in order to emphasize its non-compounding origin. Numerous numerical examples in the graphical form researching specific properties of compounding and non-compounding application contexts are presented and thoroughly analyzed. Overall results deepen significantly the understanding of the problem and provide valuable insights from this new perspective.

Yuri Shestopaloff, Ph.D.

is the Director of Technology for SegmentSoft Inc.. He is a Full Professor, Doctor of Sciences and Ph.D. in mathematical methods for data interpretation with a master's degree in Electrical Engineering. His areas of interest are developing mathematical algorithms for data interpretation and data processing in the financial industry, designing and developing middle and large-scale distributed software applications and systems, including performance measurement and trading systems. Yuri has been published in over eighty professional publications. He also published a series of articles on the philosophy of science.

Konstantin Shestopaloff

is a student at the University of Toronto specializing in Quantitative Economics. He is currently employed as a part time Quantitative Analyst at SegmentSoft Inc. He has also worked as a researcher in corporate governance at the Canadian Coalition for Good Governance, receiving mention in two publications. He has received an Honorable Mention Award for his article about Consistent Linking in The Journal of Performance Measurement in 2006.

Introduction

"…for, having lived long, I have experienced many instances of being obliged, by better information or fuller consideration, to change my opinions even on important subjects, which I once thought right, but found to be otherwise.
…Most men, indeed, as most sects in religion, think themselves in possession of all truth, and that wherever others differ from them, it is so far error."

Benjamin Franklin, from his "Speech in the Convention,
at the Conclusion of Its Deliberations"

Knowledge about life phenomena is based on comparison. Investment performance measurement is a life phenomenon by all standards. So, we should have some referral points to compare the investment portfolio and its performance. There is also another thing in this world called relativity. Whatever information and knowledge we gain, it is relative by definition. If it does not seem so, then one did not explore the whole domain of the phenomenon. This is the situation with different definitions of rates of return. We think none of them is an ultimate truthful value after we did extensive research of this problem. It depends what one is looking for, how he interprets rate of return, including his conceptual understanding of this phenomenon. The more we thought about this phenomenon, the more we were convinced that there is no ultimate cure for this problem except for the simple scenarios. Otherwise it is always tradeoff between common sense, business needs, mathematics, particular problem and its constraints and specifics, to say the least. People gravitate to simple and certain solutions just by their nature. It is nothing wrong with this quite human desire to make an order in their lives and give them peace of mind. It is widely acceptable paradigm, even if it is bought for the price of some inadequacy and subsequent consequences. So, we regret, but we have to say that however we define rate of return, it will always be a relative entity suitable within a certain context of a particular problem only. The boundaries of validity are not far away and one does not have to go really far to get some ridiculous result with the same approach. This is bad news. The good news is that methods generally work in most practical situations and produce reasonable results.

We will use results published in [Chestopalov, 2005] related to interconnection of IRR method (internal rate of return), also called MWRR (money weighted rate of return), and Modified Dietz method. It is shown in the cited work that Modified Dietz method is a mathematical derivative of IRR method. The present work explores the subject further. It will be shown that both methods beside their mathematical relationship represent two different concepts or paradigms applied toward the problem of finding rate of return for the investment portfolio.

It will be illustrated in this article that despite the relativity of all methods internal rate of return is quite stable method compared to others. It can also produce ridiculous results on some rare occasions, and we will demonstrate this by numerical examples. However, even in such occasions the source of surprise can be understood and the surprise itself can be remedied in many cases. That is situation is more or less under control when one uses IRR method. Not so with other methods. Time weighted rate of return (TWRR) as well as Modified Dietz method can surprise more and there is no reasonable explanation of some hundred thousand percent returns that can be found using these methods while the actual return is twofold the most. So, we will use IRR as a reference point below, but still we advice the reader to remember this note.

Interpretation of mathematical relationship between IRR and Modified Dietz methods

We will use IRR equation in the following form from the work cited above.

$$E = B(1+R) + \sum_{j=1}^{j=N} C_j (1+R)^{T_j} \qquad (1)$$

where T_j is time period when C_j cash flow occurred *till the end* of the total period T that is assumed to be equal one unit of time; N is the number of cash flows. All periods have to be measured in the same unit of time. Cash inflow (adding to the portfolio) is positive; cash outflow (withdrawing from the portfolio) is negative; B and E are the beginning and ending market values accordingly.

Beginning market value can be also interpreted as cash flow made at the beginning of investment period, so formally it is nothing special about this value compared to other cash flows. So, equation (1) can be rewritten as follows.

$$E = \sum_{j=0}^{j=N} C_j (1+R)^{T_j} \qquad (2)$$

where $C_0 = B$, $T_0 = 1$.

As it was suggested in the cited work, using Taylor expansion at point R = 0 equation (1) can be re written as follows.

$$E = B(1+R) + \sum_{j=1}^{N} C_j [1 + T_j R] \qquad (3)$$

Solving (3) with respect to R results in the following expression, which is Modified Dietz formula.

$$R = \frac{E - B - \sum C_j}{B + \sum C_j T_j} \qquad (4)$$

Result (4) in the described context can be viewed as a coincidence. Dietz midpoint method cannot be derived from IRR equation directly without additional assumptions. However, the relationship of IRR equation and Modified Dietz method is a remarkable fact. Modified Dietz method was a step toward the objectivity. This step was forced by the necessity in more objective valuation of rate of return. Hence the only right direction it could move into was the more accurate value of this parameter. As it was shown in work (Shestopaloff, 2007) internal rate of return is the most objective valuation of rate of return when one is using compounding approach. So, on the large scale it should not be a big surprise that the more accurate estimation of return by Modified Dietz was a step from Mid-point Dietz method toward value produced by IRR method. If not this step, there would be another one and another until at some point the next modification of this heuristic method intersects with IRR equation somehow. (People's nature quite often explores dead ends as well, but practical needs return them on the right track eventually in *all* such situations.) So, the event of turning to IRR direction has been predestined from the very beginning once the concept of rate of return originated. It just happened that a particular intermediate form turned out to be Modified Dietz equation.

Geometrical interpretation of Modified Dietz equation in relation to IRR function

Now we would like to give geometrical interpretation of our finding about Modified Dietz method as a linear approximation of IRR equation. We drew two IRR functions $E = F_1(R)$ and $E = F_2(R)$. We discovered before that Modified Dietz method is a linear approximation of IRR equation at point $R = 0$, that is just a tangent to IRR functions at points with coordinates $(0, F_1(0))$, $(0, F_2(0))$. The solution Modified Dietz equation provides is value R_{MF}, (MF denotes Modified Dietz) the value of rate of return at which linear function t_1 intersects the line $F(R) = EMV$. This rate of return differs from rate of return found by IRR method R_{IRR} . Similarly rates of return differ for the IRR function $F_2(R)$ and tangent t_2 implementing appropriate linear Modified Dietz function for IRR function $F_2(R)$.

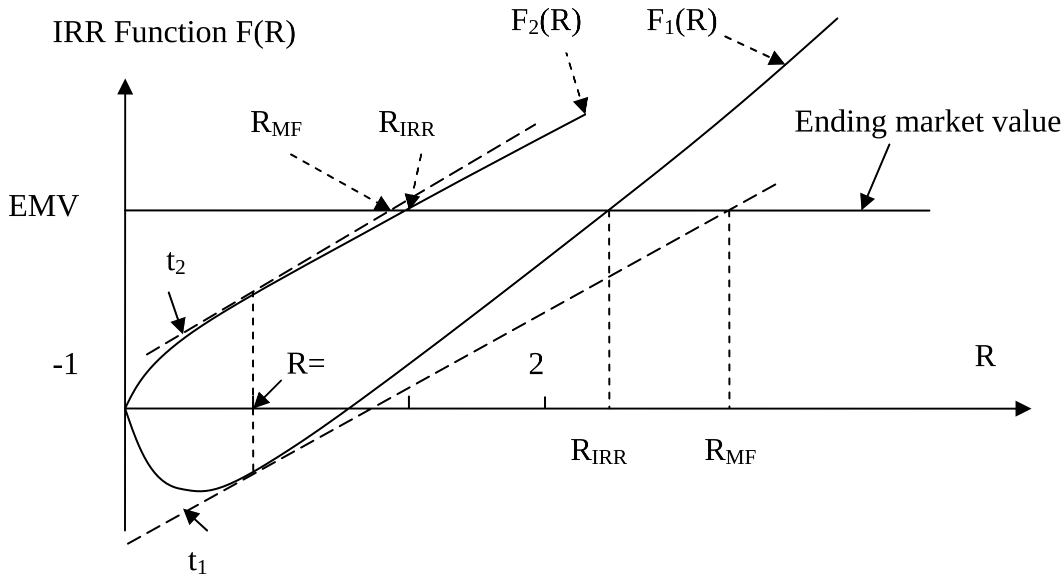

Fig. 1. Graphical interpretation of Modified Dietz equation as a linear function (tangent to IRR function at point R = 0). R_{MF} and R_{IRR} are rates of return corresponding to Modified Dietz method and IRR equation accordingly.

We would like to point the reader's attention to the situation when tangent t_1 is drawn to the left of function's $F_1(R)$ minimum (we can create such a combination of beginning market value and cash flows when this situation is possible). If this is the case, then Modified Dietz equation will produce value of return less then (-1), that is invalid solution. This is one of the limitations of this approach.

Compounding context of IRR equation

At the beginning of this article we made a note regarding relativity of our knowledge about complex problem finding objective value of rate of return. Now as the reader acquainted formal knowledge of mathematical interrelations of two considered methods, we can climb higher to the conceptual level of this absolutely not trivial problem. We start from discussion of compounding and non-compounding concepts inherently incorporated into this subject, though never considered as an important ingredient. In reality this factor is of outmost importance. It defines the most essential characteristic of methods for calculating rate of return that we call as compounding and non-compounding context of application.

IRR equation (1) by its nature is a compounding equation. So, when we moved to the realm of rate of return thus we inherently included compounding context into our considerations. But if this assumption about compounding context is always justified? Think about such a situation. Someone bought shares and keeps them in the portfolio. Shares' price goes up and down all the time, maybe growing on average for years thanks to inflation. What kind of compounding do we really face in this situation if we decide to apply IRR equation to measure ours investment success? Maybe we do not have compounding at all. If we did not touch our portfolio all these long years full of anxiety and investment uncertainty, then simple rate of return will be a good measure to us, that is R = (E – B)/B, where E and B are accordingly ending and beginning market values. Suppose we invested for ten years. Then yearly rate of return is what? Is it $R_y = (1+R)^{\frac{1}{10}} - 1$ or just R/10? There is no easy answer to this question, really. If we pick up the first answer, then this implies compounding, which is not a certainty. Then we left with the second non-compounding choice which is not the obvious one too. So, in this case this is a matter of agreement and chosen mathematical model. Actually both answers are true within their respective mathematical model, though both models are not exactly the adequate equivalents of the reality. The problem is that we do not have criteria what approach to be applied to this problem. The situation is even more interesting, because we cannot have such unambiguous criteria principally in this case – not enough certainty.

Compare, for example, annuity. One invests the same amount of money on a regular basis. Everything is known – amount, payment schedule, interest rate to be applied and when. Investment portfolio, on the other hand, looks more like a black box. We know what input we provided, what outcome we received and maybe what cash transactions we made and when. The rest is murky. Maybe our money was idle almost for the whole investment period and grew only for the last five days, or vice versa – we do not know. It means that when we use compounding approach in this situation, it is actually quite arbitrary decision. Shares do not increase its price because of the previous increase, not to say do this systematically and strictly proportionally as annuities do. It is not unlike sending child to remote college without communication. We can equally assume he is doing great and work really hard crunching granite of science or, he does not attend classes at all. Even if in five months the child is no longer with in the college, we still do not know either he worked hard but not smart enough for this college, or he is bright but undisciplined person incapable to organize his work without parents' supervision. Same situation is when investor gets the final result.

Distinguishing direct and reverse problems

Beside the inherent uncertainty of the investment performance valuation there is another important consideration. In the core the described problem is that we are solving *reverse* problem using approaches and concepts developed for the *direct* problem, when everything is known, like in the mortgage arrangement. Formulation of reverse and direct problems is principally different because we cannot apply conditions and restrictions of direct problem toward reverse problem automatically. The last one has different context. However, we do not have methods developed in the context of reverse problem and specifically for it, how strange it may sound. Some existing approaches demonstrate this statement.

Let's take TWRR (time weighted rate of return). Essentially this is heuristic algorithm within the domain it is applied to. It did not originate to serve this domain. Its foundation is geometric linking operation. Geometric linking is valid only for sequential periods when beginning market value of every next period is equal exactly to the ending market value of the previous period. (We provided a strict mathematical proof of this statement in the form of Geometric Linking Theorem that we intend to publish in a separate article.) Otherwise geometric linking is doomed to produce incorrect value. Nonetheless, in TWRR case it is applied to linking rates of return for periods separated by cash transactions, hence a priori and specifically outside the legitimate domain of applicability of geometric linking. What does it mean within the context of our considerations? It means that geometric linking is an alien method to the problem of finding rate of return for investment portfolio.

We would like to emphasize another feature of TWRR and geometric linking. Time of investment does not influence the total return. Suppose we link two periods with a total length ten days. Neither beginning nor market value of periods are changed. Cash inflow is also the same. What is different is the length of periods composing the whole period. However, whatever combination of period lengths is used, total return will be always the same. Is it such insensitivity good or bad thing? We do not know. It depends on the situation. Modified Dietz return and IRR take into account time when cash flow occurs. It can make a big difference to the outcome these methods produce even if all other values are equal. So, these methods implicitly answer the question *how* this return has been acquired besides answering the question *what* return this particular portfolio delivered. Combining two characteristics into one number sometimes might be good, sometimes not. Again, depends what we are looking for. Generally in such situations two separate characteristics deliver overall picture of a better quality.

We mentioned already that IRR includes compounding implicitly, just because this is the nature of IRR equation. This is perfectly legitimate approach when one solves *direct* problem, let's say considering mortgages or annuities. Compounding legitimacy is not so obvious when we look from the *reverse* side. This is exactly what the problem of finding rate of return is about, as we tried to explain above our vision of this problem.

Non-compounding nature of Modified Dietz equation

Let's assume for a moment that the problem finding rate of return has a *non-compounding* context while we still want to assess the influence of cash flows' transaction time onto rate of return. We assume that period length is equal to T_0 units of time. Then transaction time for cash flow done at the very beginning of period is equal to T_0, while transaction time for cash flow made at the very end of investment period is equal to zero. Non-compounding context means that rate of return to be applied toward cash transaction is proportional to time period this cash remains within the portfolio. Using our previous notations we can rewrite this assumption in mathematical form as follows.

$$E = \sum_{i=0}^{i=N} C_i (1 + RT_i) \qquad (5)$$

where T_i is time period from cash flow occurrence till the end of the period measured in *arbitrary* units of time which are the same for all cash transactions; R is rate of return for a one chosen unit of time.

Solving this equation with respect to R produces the following equation:

$$R = \frac{E - \sum_{i=o}^{i=N} C_i}{\sum_{i=o}^{i=N} C_i T_i} \qquad (6)$$

As a particular case we can consider values $C_0 = B$, and assume $T_0 = 1$. Then equation (6) becomes nothing else but somewhat modernized Modified Dietz formula. This is not exactly a surprise, because Modified Dietz formula's linearity always raised the question about its compatibility with compounding context. However, it was not proved explicitly until now. The fact that a particular case of equation (6) produces Modified Dietz formula should not mask its novelty as equation for non-compounding calculation rate of return. We are not sure that we should use some distinguishing name for this formula. This may look as a too pretentious act. At the same time the inner qualitative content of this equation is quite different from Modified Dietz equation. The last one completely ignored compounding and non-compounding context and even did not suspect the mere existence of these issues. Secondly, Modified Dietz equation could not be used without the assumption that the period length is equal to *one* unit of time. Thus the obtained rate of return by silent and never discussed default assumption can be applied only to this whole period that always has one unit of length. Were no ways to recalculate this return onto smaller or bigger period. Equation (6) allows to do this naturally and *correctly* both from business and mathematical perspectives using well defined linear dependence of rate of return on the period length, which is the consequence of its non-compounding context. These are essential and principal differences. So, we are inclined to think of this equation as somehow special, let's say *generalized Modified Dietz equation* in order to distinguish its non-compounding context and widen the area of applicability. Reader should understand that such a coincidence just happened because of a few contributing factors, including bright mind of authors of Modified Dietz formula. Otherwise equations could be different. Suppose, if we did this derivation prior few decades, then we would compare equation (6) to mid-point Dietz equation instead of Modified Dietz formula. We used no knowledge of Modified Dietz equation and none of the assumptions its derivation was based upon. These are completely independent entities how weird it may sound to some readers. It is an understandable opinion, the equations are so similar to each other. Nonetheless, it is not so if one looks deeper.

This is actually a very good result. Knowing that we deal with non-compounding context when using Modified Dietz formula grants a great deal of certitude and deepens our understanding what we are really doing crunching the numbers with Modified Dietz formula. Beside, *generalized Modified Dietz formula* enhances the scope its applicability and adds more functionality, allowing calculating rate of return for a period with arbitrary length and recalculating the result to rate of return for a period with different length. Thus we found one more connection to reality and discovered new features of Modified Dietz equation, this time as an instrument for finding non-compounding rate of return. On the opposite side we have IRR equation with its purely compounding context. However, both approaches are united under the umbrella of idea to provide rate of return combining answers to two questions: *what* is the value of rate of return and *how* this accomplishment has been achieved meaning the influence of cash flows.

What about geometric linking, the foundation of TWRR with respect to compounding context? The reader can easily answer this question himself. Right, obviously geometric linking is a compounding entity. If rates of return for all periods are equal and ending market value of each period is equal to ending market value of the previous period, then it becomes a classic compounding equation. There is a “minor” inconvenience with this approach that it is applied to problem located completely outside the domain of its applicability.

We reiterate one more time that TWRR produces rate of return essentially independent of cash flows disregarding the question “how” appealing to Modified Dietz and IRR methods. This is an important conceptual feature of geometric linking and TWRR method wryly derived from it.

Numerical examples demonstrating conceptual differences between Modified Dietz and IRR methods

We will explore the following approach to research the issue of compounding and non-compounding contexts with regard to rate of return. Our simple investment portfolio includes one or two cash transactions. We will change only time when cash transactions occur, leaving the rest of portfolio’s parameters untouched, see Fig. 2. We will measure transaction time as a fraction of time from the moment cash transaction occurred till the end of investment period. It means that if cash transaction done at the beginning of period, then time of transaction is equal to one unit of time.

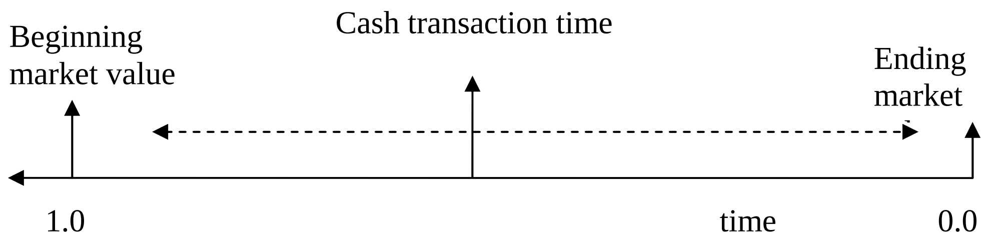

Fig. 2. Sample portfolio with variable cash transaction time.

Next Fig. 3 illustrates the change experienced by IRR and Modified Dietz return when cash transaction time is changing. Beginning market value is \$100, cash transaction is \$100, ending market value is \$500. We will use the following notation below to denote these values: P={100, 100, 0}.

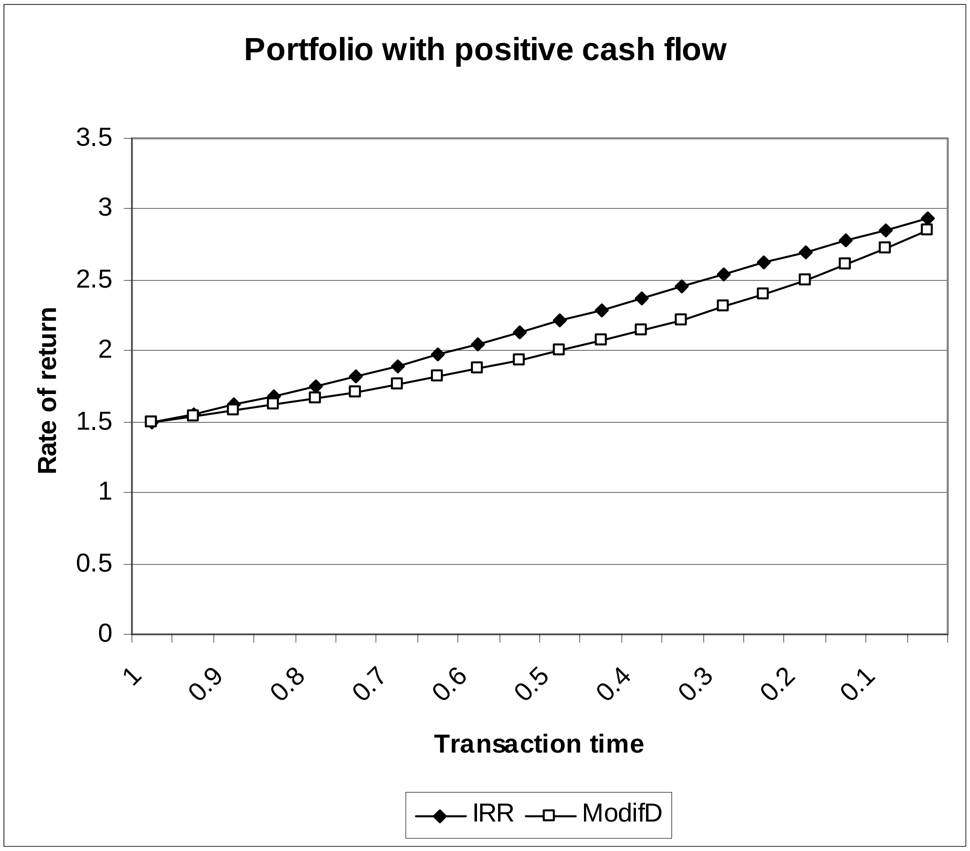

Fig. 3. Modified Dietz and IRR returns depending on transaction time. P={100, 100, 0}, EMV=500.

We can see from Fig. 3 that when transaction time is increasing (cash transaction has been added to the portfolio closer to the period's beginning), then IRR is decreasing. Intuitively this is the right thing to do. What happened? We have *positive* cash flow. The longer this positive cash flow stays within the portfolio, the more interest it will accrue. It means to us that the smaller rate of return is required for infused cash with bigger investment period to accumulate the same increase needed to reach the same fixed ending market value of the portfolio. Also, please do not forget that beginning market value accumulates some interest as well. So, the resulting rate of return is a fine balance between portfolio's increase provided by cash flow and beginning market value and the value of rate of return. Smaller transaction time means that cash addition has less time to grow and beginning market value has to work harder to accumulate bigger increase. As a result of these two factors the rate of return increases. Similar growth of interest rate accompanies rate of return computed with Modified Dietz formula when transaction time decreases. However, Modified Dietz return grows slower compared to IRR. This phenomenon is explained by the next Fig. 4. IRR function presented on this figure increases slower than appropriate dependence for a Modified Dietz formula. In mathematical terms we are talking about growth of polynomial function with power less than one unit versus linear function, that is $(1+R)^{T_j}$ versus $(1+RT_j)$. It can be seen from this figure that internal rate of return has to be slightly higher in order to provide the same ending market value.

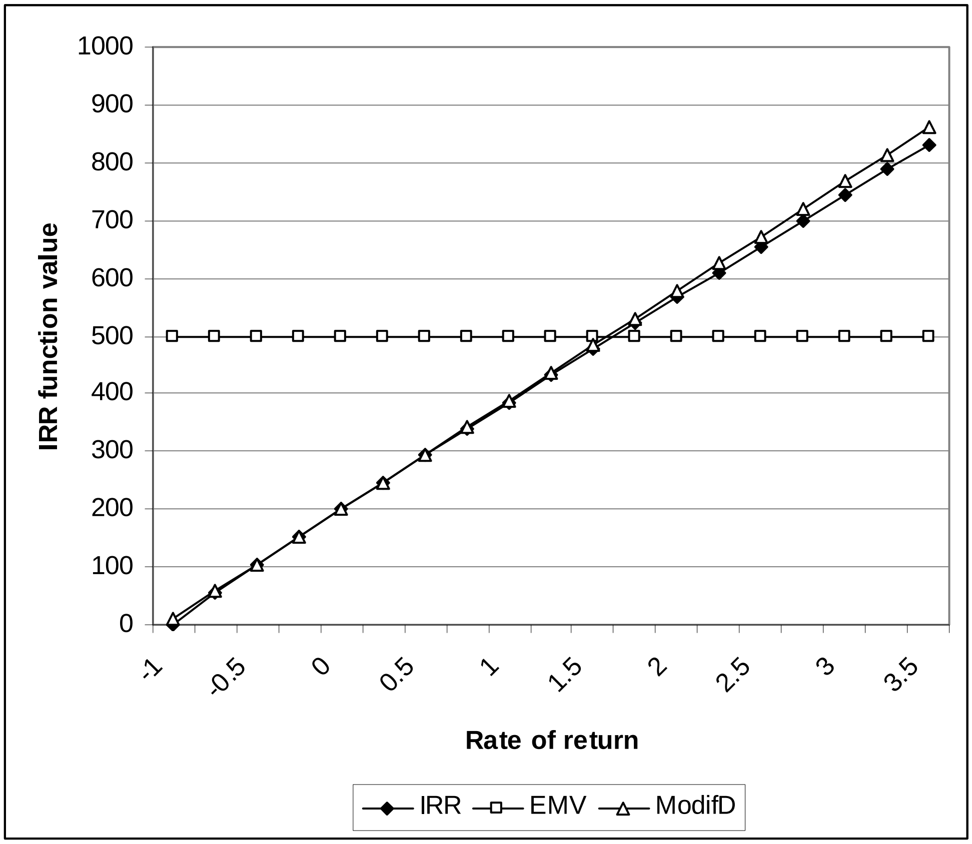

Fig. 4. IRR function and linear dependence defined by Modified Dietz equation.

A note regarding time scale conversion.

We agreed that period length is one unit. If this is not the case, then the following conversion has to be made to adjust obtained rate of return toward the whole period. For example, if the total period length is T units, then return for the whole period is defined according to compounding context of IRR equation as follows:

$$R_{IRRtotal} = (1+R)^T - 1, \qquad (7)$$

while Modified Dietz return to be multiplied by the period length because of its non-compounding linear nature:

$$R_{MDtotal} = RT \qquad (8)$$

Here, again, we face essentially conceptual question. Both assumptions about compounding and non-compounding contexts of the problem are legitimate because neither one has any inherent relationship with the actual change of portfolio's value. It is to some extent is similar to measuring some quantity of liquid. We can do it in pounds or in gallons, but one is unit of weight, the other one is unit of volume. Both are perfectly legitimate units of measure for our situation. Which one would you prefer? If you say gallons, we can object to your wise and highly appreciated choice that the liquid might have a very high temperature coefficient expanding twice per every ten degrees increase in temperature. So, we argue, it might be not the best approach. If you prefer pounds, we might say that we have no idea what liquid it is, what density it has. So, if we want to transport it to another location, this preference will not help us to find out, what is the volume capacity of the tank the carrier should use to do the job. You see, life is always about pros and cons and about choice. Preferably the wise choice. So, phase one of the competition did not discover the winner and we should use more considerations and analyze more pertinent factors.

Let's consider portfolio with negative cash flow. P={200, -100, 0}, ending market value EMV=500. Fig. 5 presents appropriate graphs for IRR and Modified Dietz returns.

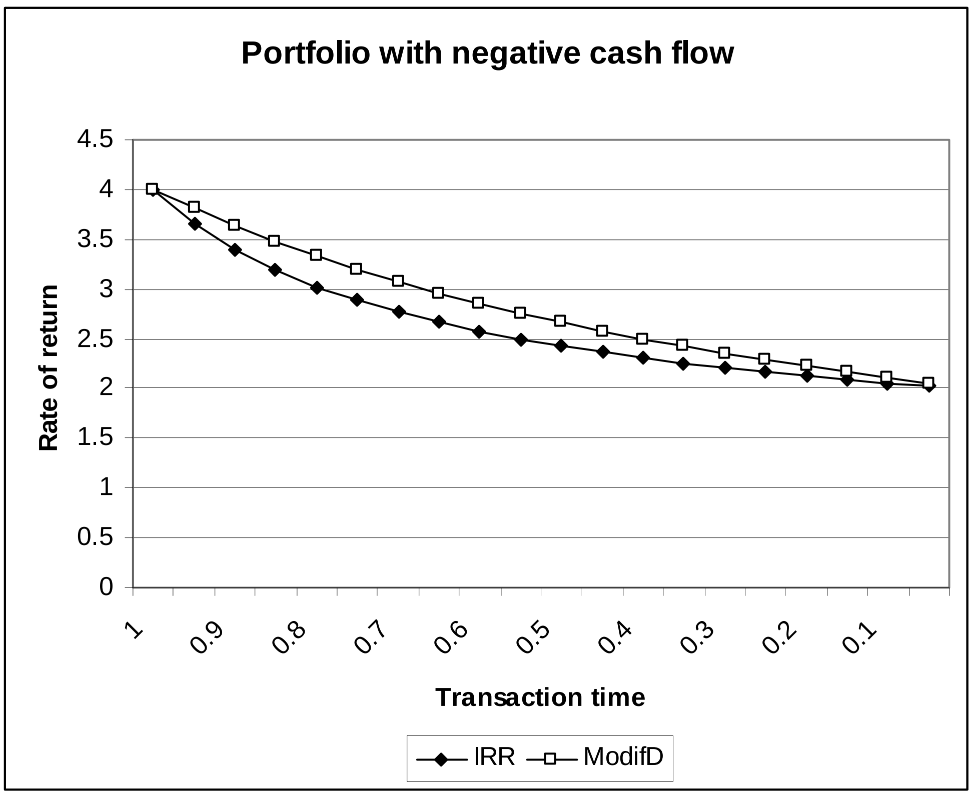

Fig. 5. IRR and Modified Dietz returns for a portfolio with negative cash flow depending on transaction time.
P = {200,-100, 0}.

Both rates of return decrease when transaction time decreases, the behavior opposite to portfolio with positive cash flow. This is due to the fact that the shorter time period the negative cash flow stays in the portfolio, the less impact it provides onto portfolio's performance. If it is done at the period's beginning, then negative cash flow works against the beginning market value. Accordingly the last one has to provide higher growth in order to secure the fixed ending market value.

Graph of Modified Dietz dependence this time goes higher than graph for IRR, that is also right to the contrary to the situation on Fig. 4. We resort to graphs of IRR and linear (Modified Dietz) functions in order to interpret the results, see Fig. 6.

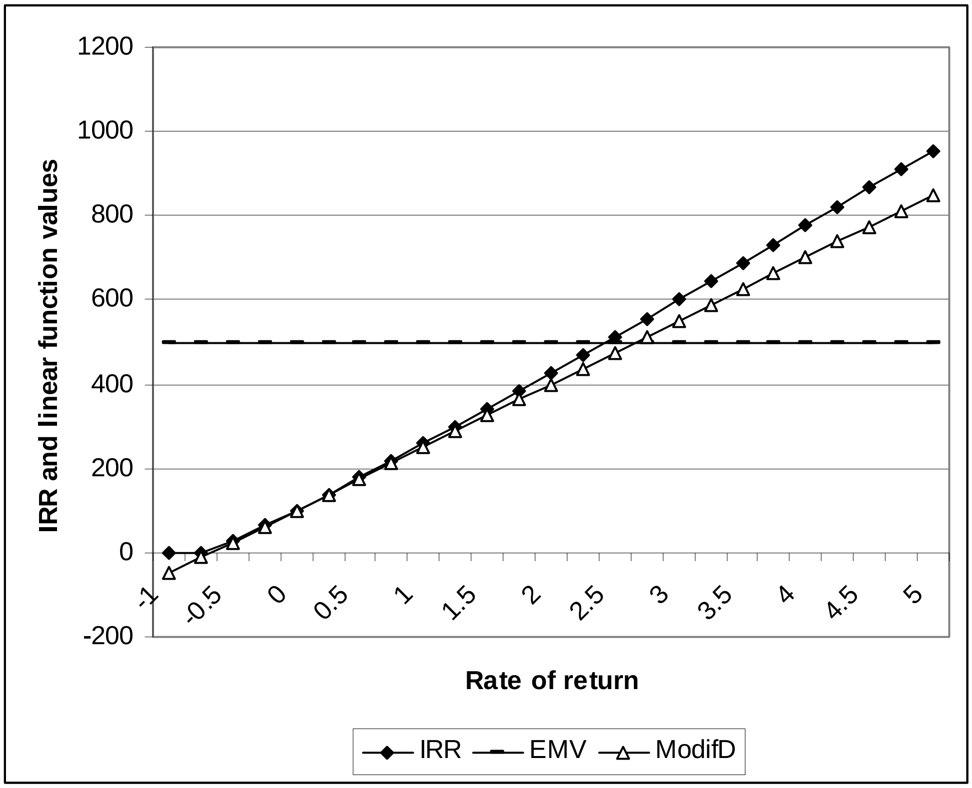

Fig. 6. IRR and linear (Modified Dietz) functions' values depending on rate of return.

Let's take a look at Fig. 6. We see that graph of IRR function goes above the linear Modified Dietz function. We remember from studying the case of positive cash flow that compounding produces smaller absolute growth for the same rate of return. In the case of negative cash flow it means that smaller value is *subtracted* from portfolio value, this time provided by the beginning market value only. Modified Dietz linear function provides bigger (in absolute value terms) decrement caused by cash flow, thus subduing total Modified Dietz linear function more significantly. Accordingly the value of IRR function is higher and its graph is above the graph of Modified Dietz function. With respect to Fig. 5 it means that Modified Dietz approach requires higher rate of return than IRR in order to provide the same ending market value.

Introducing second transaction changes the pattern slightly. Fig 7 demonstrates this relative to portfolio with two cash flows. This time the overall behavior more complicated. It is possible to perceive the reasons for that too, as we did for portfolio with one cash flow. However, that is not so crucial to us. We just have to get an idea that permutations of different cash flows produce noticeable but not very easy predictable changes in rates of return defined by two analyzed methods. More or less balanced amounts and number of inflow and outflow cash transactions smoothes and makes closer the curves. Single sign transactions, on the opposite, increase the distance between the two curves.

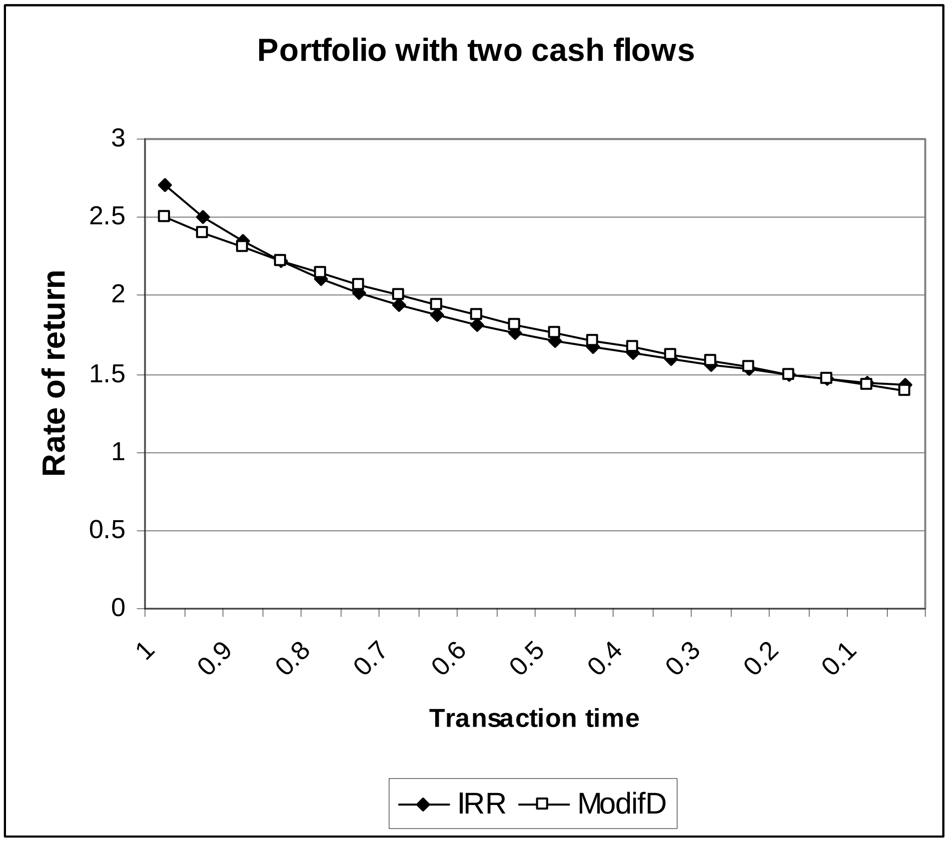

Fig. 7. Portfolio with two cash flows depending on transaction time of the first cash flow. Transaction time of the second cash flow is equal to 0.2. P={200, -100, 100}, EMV=500.

Limitations of IRR method and Modified Dietz equation

Now we will create the following extreme portfolio to explore the limitations of our approaches. Let P={200, -250, 0} with EMV=500. The graph for IRR and Modified Dietz returns are presented on Fig. 8.

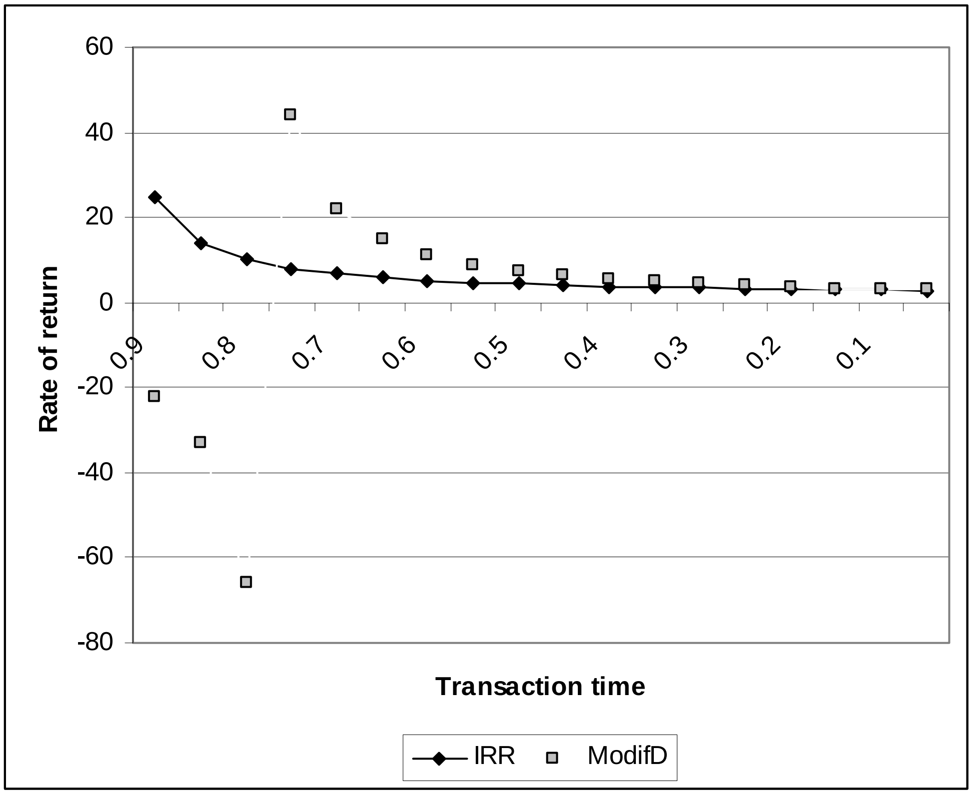

Fig. 8. Rate of return depending on the transaction time for the extreme negative cash flow.

We can see that Modified Dietz dependence has a break near the point T = 0.8. The left branch goes to negative infinity, the right branch goes to positive infinity. Obviously the left branch has no practical meaning to us, as well as extremely high values of return on the right branch. This happens because the denominator of Modified Dietz formula becomes equal to zero. Internal rate of return behaves much more reasonably providing meaningful values of return for the whole range of transaction time. The rate of return just has to grow really fast in order to compensate the large negative cash flow. When transaction time decreases, both returns tend to come closer. This fact means that the influence of cash flow decreases when the time period it stays in the portfolio shortens.

So, Modified Dietz approach apparently has limitations compared to IRR approach. Its denominator should not be close to zero and sum of products of cash flows by transaction time should not be such that it is negative and its absolute value exceeds the beginning market value. We will not elaborate any further recommendations regarding applicability of Modified Dietz formula. There can be more of them. What we think it is reasonable in such a situation to check Modified Dietz return against IRR value. If these numbers have the same sign and close to each other in the range less than several tenths of a percent, then probably we are in the right range for this particular portfolio and can do further adjustments. The other consideration to be noted is that large transactions near the period's beginning affects the portfolio's characteristics like rate of return significantly.

Concluding two previous sections we would like to summarize its results as follows.

1. Both Modified Dietz approach and internal rate of return (also called as money weighted rate of return) are legitimate approaches for the purpose of evaluation rate of return on investment portfolio. The first one provides non-compounding context, the second creates compounding context. Accordingly methods produce non-compounded and compounded rates of return.
2. Investment portfolio is essentially a black box hiding the process and mechanism of changing market value of the investment portfolio. It implements neither compounding mechanism, nor non-compounding exactly. It can include both in different proportions or none of them. Investment portfolio's market value can grow actually in unpredictable and unexpected manner. This is why the instruments we apply to investment portfolio do not have direct relationship with the phenomenon of market value changing itself**. These instruments are foreigners brought from other domain to do the job they were not created for.**
3. Both methods have their own specifics and limitations. However, internal rate of return is a more stable and adequate instrument for measuring investment performance because of its predictable and readily interpreted results. At the same time, it should be understood that IRR always exists within a compounding context, the context it initially originated from. Modified Dietz method, on the other hand, has to be always associated with strictly non-compounding context and appropriate application.
4. Discussed methods both incorporate into the number they produce two characteristics of the investment portfolio. The first reflects the influence of cash flow values and transaction time when this cash has been added to the portfolio or withdrawn from it. The second characteristic, the rate of return itself, is an integral parameter evaluating portfolio as a whole entity, defining its performance. It might be seen as a subtle issue, but in reality it is the important one. For example, TWRR produces only rate of return and by no means takes into account cash flows and its transaction time.

Completing discussion on conceptual differences between methods for calculating rate of return

We learned that Modified Dietz method is a mathematical derivative of IRR method. It turned out that it is a particular approximation of IRR equation. At the same time it is self-sufficient method in its realm of non-compounding application context. As a result we have an interesting situation. On the practical, business side each method is associated with its own conceptual paradigm non-intersecting with the one of another method. On the other hand methods are tied by inherent mathematical interrelation. **If this is contradicting situation or not, and if so, what view one should prefer?** We do not see much contradiction or reasons for confronting business and mathematical aspects. In reality these aspects interrelate exactly to the extent how compounding and non-compounding approaches relate to each other because this is the only core reason of the divergence between methods we observe. As we discussed already in length, given the “black box” nature of investment portfolio from performance perspective, both approaches are perfectly legitimate tools for the job. We can use mallet, or hammer or sledge to crack the nut, can’t we? It is a question of what tool is available to us or, if we have some, what is the most appropriate one for the particular nut.

Mathematically Modified Dietz method and IRR approach are closely interconnected through the relation of its appropriate non-compounding and compounding contexts. So, from the mathematical perspective Modified Dietz is an approximation of IRR method when compounding context can be approximated by non-compounding one. This lays down strong case for unifying numerous performance measurement methods toward fewer standards based on business application. Eventually it will make the performance measurement business more standard, objective and streamlined.

Presented research showed that internal rate of return should have a more advanced role in the performance measurement because of its better stability and objectivity within the compounding context. It is also a mathematical parent of other used methods presently considered as independent investment valuations. This point of view is justified to some extent in the business sense as we demonstrated in this article for Modified Dietz method and IRR approach.

IRR equation can be easily accommodated to calculate rates of return for multiple periods within a single reporting period in order to monitor the dynamics of investment process. Thus it can serve many more business scenarios. Given the computing power of modern enterprises, even the small ones, to exercise such approach by numerically solving IRR equations is not an issue anymore. So, IRR method should be given an adequate respect and actually a leading role in the hierarchy of methods used for calculating rates of return as an objective criteria of the investment process. Saying this we by no means imply that every application has to stick to IRR equation, even in the compounding context. Life is much more diverse than a single equation can propose and describe. What we are trying to say is that IRR equation is a solid instrument within its applicability domain and as such it can be used to develop other tools and instruments to solve new problems, inevitably emerging as life goes on.

Proposed methods could create foundation for the unification of performance evaluation standards. It is crucially important that IRR and other methods in use have to be defined within appropriate contexts and applicability domains. In particular, it can be done by using direct relationship of IRR with other methods. At present it seems like none of other methods can challenge the IRR’s supremacy within compounding context if there is a task to get just one number. However, this requirement can change in the due course of life.

As long as we manage to wrap things up and find some universal approach or concept uniting things assumed before as different creatures, we are victorious. Seriously. Introducing many things, terms and notations is a relatively easy task. It is a great and rare skill to see generality and wrap complex diverse phenomenon into compact and elegant entity embracing existing phenomena and providing new bewildering insights. Without this skill science would degenerate into pile of descriptions and loosely related terms and notions, at best. The real progress is about wrapping thing up to the point of creating new advanced quality. This is ongoing process, all the time, at every step. This is *the only* way to move knowledge to a new qualitative level. Introduced notion of compounding and non-compounding contexts for rate of return serves to such wrapping purpose to some extent uniting things in reality more related than we used to think of before.

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